

Time: 3 Hours

Total Marks: 80

N.B.

- 1) Q.1 is compulsory.
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

- a) Prove using Mathematical Induction that $2 + 5 + 8 + \dots + (3n-1) = n(3n+1)/2$ **5**
- b) Explain the term poset. Consider a set D_{165} . Find the elements of this set & draw the hasse diagram for this poset. **5**
- c) How many strings of length 7 either begin with 2 zeros or end with 3 ones? **5**
- d) Explain the term partition set with suitable example. **5**
- e) State the Pigeonhole principle and show that If there are 10 marbles in the jar & you have a jar filled with red, green, and blue marbles, you'll always have at least two marbles of the same colour. **5**

Q.2

- a) **10**
 Let $A = \{0, 1, 2, 3, 4, 5\}$
 i) Explain the term group.
 ii) Prepare the composition table for the above set w.r.t. the operation of addition modulo 6.
 iii) Determine whether it is a group.
 iv) Whether elements of set A are invertible? If yes, then find the inverses of these elements.
 v) Determine whether it is a cyclic group.

b

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and let R be a relation on A whose matrix is :

10

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find out transitive closure of R using Warshall's algorithm.

Q.3

- a) A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.

Determine the number of computer programmers that are not proficient in any of these three languages. **8**

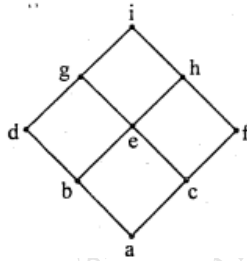
b) Explain the terms Conjunctive & Disjunctive Normal Form with suitable examples. **6**

c) **6**

Determine the sequence b_n whose recurrence relation is $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$.

Q.4

a) What is a lattice? Determine whether following hasse diagram represents a lattice. **6**



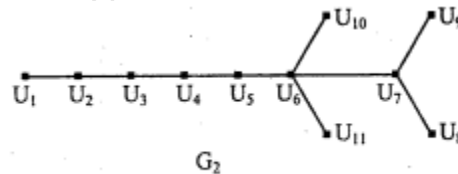
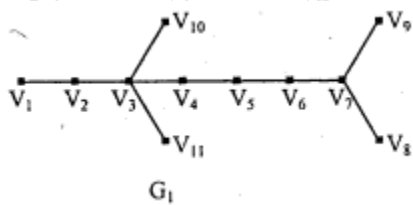
b) **6**

Consider $(3, 8)$ an encoding function $e : B^3 \rightarrow B^8$ defined as

- $e(000) = 00000000$
- $e(001) = 10111000$
- $e(010) = 00101101$
- $e(011) = 10010101$
- $e(100) = 10100100$
- $e(101) = 10001001$
- $e(110) = 00011100$
- $e(111) = 00110001$

How many errors can 'e' detect & correct?

c) What are the necessary conditions for the isomorphism between 2 graphs? Determine whether following 2 graphs are isomorphic. **8**



Q.5

a) If the addition & multiplication modulo 10 is defined on a set of integers $A = \{0, 2, 4, 6, 8\}$. Then determine whether this algebraic system is a ring. **8**

b)

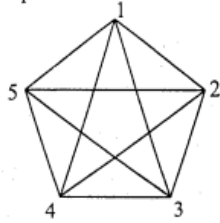
A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$
 Is it i) injective ii) surjective iii) bijective

6

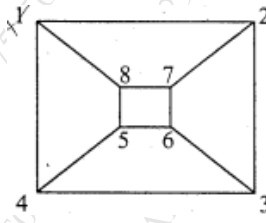
c) Define the terms Euler path & a circuit .

6

Determine whether following graphs have Euler path or a circuit.



a)



b)

Q.6

a) Explain the following terms with suitable example (any 4)

8

- i) Hamiltonian path & circuit
- ii) Bipartite graph
- iii) Adjacency matrix
- iv) Equivalence relation
- v) Cartesian product

b) Solve the following using the laws of logic

4

$$p \vee q \vee (\sim p \wedge \sim q \wedge r) \leftrightarrow p \vee q \vee r$$

c)

8

$f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^3$
 $g: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = 4x^2 + 1$
 $h: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $h(x) = 7x - 2$
 Find the rule defining

- i) fog ii) gof
- iii) (goh)of iv) go(hof)